Section 5.3 ELECTROMAGNETIC RELATIONS

	Gaussian CGS	SI
Units and conversions:		
Charge	$2.997\,92 \times 10^{9}$ esu	$= 1$ C $= 1$ A s
Potential	(1/299.792) statvolt = (1/299.792) erg/esu	$= 1$ V $= 1$ J C^{-1}
Magnetic field	10^4 gauss = 10^4 dyne/esu	$= 1$ T $= 1$ N A^{-1} m^{-1}
Electron charge	$e = 4.803\,204 \times 10^{-10}$ esu	$= 1.602\,176 \times 10^{-19}$ C
Lorentz force	$\mathbf{F} = q\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right)$	$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$
Maxwell equations	$\nabla \cdot \mathbf{D} = 4\pi\rho$	$\nabla \cdot \mathbf{D} = \rho$
	$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0$	$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$
	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
	$\nabla \times \mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} = \frac{4\pi}{c}\mathbf{J}$	$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$
Linear media	$\mathbf{D} = \varepsilon \mathbf{E}$, $\mathbf{B} = \mu \mathbf{H}$	$\mathbf{D} = \varepsilon \mathbf{E}$, $\mathbf{B} = \mu \mathbf{H}$
Permittivity of free space	$\varepsilon_{\text{vac}} = 1$	$\varepsilon_{\text{vac}} = \varepsilon_0$
Permeability of free space	$\mu_{\text{vac}} = 1$	$\mu_{\text{vac}} = \mu_0$
Fields from potentials	$\mathbf{E} = -\nabla V - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$	$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$
	$\mathbf{B} = \nabla \times \mathbf{A}$	$\mathrm{B} = \nabla \times \mathrm{A}$
Static potentials (coulomb gauge)	$V = \sum_{\text{charges}} \frac{q_i}{r_i}$	$V = \frac{1}{4\pi\varepsilon_0} \sum_{\text{charges}} \frac{q_i}{r_i}$
	$\mathbf{A} = \frac{1}{c}\oint \frac{I\,\mathbf{d}\ldots}{\lvert \mathbf{r} - \mathbf{r}' \rvert}$	$\mathbf{A} = \frac{\mu_0}{4\pi}\oint \frac{I\,\mathbf{d}\ldots}{\lvert \mathbf{r} - \mathbf{r}' \rvert}$
Relativistic transformations (**v** is the velocity of primed system as seen in unprimed system)	$\mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel}$	$\mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel}$
	$\mathbf{E}'_{\perp} = \gamma\left(\mathbf{E}_{\perp} + \frac{1}{c}\mathbf{v} \times \mathbf{B}\right)$	$\mathbf{E}'_{\perp} = \gamma\left(\mathbf{E}_{\perp} + \mathbf{v} \times \mathbf{B}\right)$
	$\mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel}$	$\mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel}$
	$\mathbf{B}'_{\perp} = \gamma\left(\mathbf{B}_{\perp} - \frac{1}{c}\mathbf{v} \times \mathbf{E}\right)$	$\mathbf{B}'_{\perp} = \gamma\left(\mathbf{B}_{\perp} - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}\right)$

$$4\pi\varepsilon_0 = \frac{1}{c^2}\,10^{7}\ \mathrm{A^2\ N^{-1}} = \frac{1}{8.987\ 55\ldots} \times 10^{-9}\ \mathrm{F\ m^{-1}}$$

$$\frac{\mu_0}{4\pi} = 10^{-7}\mathrm{N\ A^{-1}}\ ;\ c = 2.997\ 924\ 58 \times 10^{8}\ \mathrm{m\ s^{-1}}$$

Impedances (SI units)

ρ = resistivity at room temperature in $10^{-8}\ \Omega$ m:

~ 1.7 for Cu	~ 5.5 for W
~ 2.4 for Au	~ 73 for SS 304
~ 2.8 for Al (Al alloys may have double this value.)	~ 100 for Nichrome

For alternating currents, instantaneous current I, voltage V, angular frequency ω:

$$V = V_0 e^{j\omega t} = ZI \ .$$

Impedance of self-inductance L: $Z = j\omega L$.

Impedance of capacitance C: $Z = 1/j\omega C$.

Impedance of free space: $Z = \sqrt{\mu_0/\varepsilon_0} = 376.7\,\Omega$.

High-frequency surface impedance of a good conductor:

$$Z = \frac{(1+j)\rho}{\delta} \text{ , where } \delta = \text{effective skin depth ;}$$

$$\delta = \sqrt{\frac{\rho}{\pi\nu\mu}} \cong \frac{6.6 \text{ cm}}{\sqrt{\nu[\text{Hz}]}} \text{ for Cu .}$$

Capacitance $\hat{C}$ and inductance $\hat{L}$ per unit length (SI units)

Flat rectangular plates of width w, separated by $d << w$ with linear medium (ε, μ) between:

$$\hat{C} = \varepsilon\frac{w}{d} \ ; \quad \hat{L} = \mu\frac{d}{w} \ ;$$

$$\frac{\varepsilon}{\varepsilon_0} = 2 \text{ to } 6 \text{ for plastics; } 4 \text{ to } 8 \text{ for porcelain, glasses;}$$

$$\frac{\mu}{\mu_0} \cong 1 \quad .$$

Coaxial cable of inner radius r_1, outer radius r_2:

$$\hat{C} = \frac{2\pi\varepsilon}{\ln(r_2/r_1)} \ ; \quad \hat{L} = \frac{\mu}{2\pi}\ln(r_2/r_1) \ .$$

Transmission lines (no loss):

Impedance: $Z = \sqrt{\hat{L}/\hat{C}}$.

Velocity: $v = 1/\sqrt{\hat{L}\hat{C}} = 1/\sqrt{\mu\varepsilon}$.

Motion of charged particles in a uniform, static magnetic field

The path of motion of a charged particle of momentum p is a helix of constant radius R and constant pitch angle l, with the axis of the helix along **B**:

$$p[\text{GeV}/c]\cos\lambda = 0.29979\, qB[\text{tesla}]\, R[\text{m}]\ ,$$

where the charge q is in units of the electronic charge. The angular velocity about the axis of the helix is

$$\omega[\text{rad s}^{-1}] = 8.98755\times 10^{7}\, qB[\text{tesla}]/E[\text{GeV}]\ ,$$

where E is the energy of the particle.

This section was adapted, with permission, from the 1999 web edition of the *Review of Particle Physics* (http://pdg. lbl.gov/). See J. D. Jackson, *Classical Electrodynamics*, 2d ed. (John Wiley & Sons, New York, 1975) for more formulas and details.